4.3 – Lines in Motion

Daily Objectives:

1. Describe translations of a line in terms of horizontal and vertical shifts
2. Write the equation of a translated line using h and k
3. Understand point-slope as a translation of a line written in intercept form
4. Apply translations to functions

> **Translation of a Function**
>
> A translation moves a graph horizontally or vertically or both.
>
> Given the graph of $y = f(x)$, the graph of
>
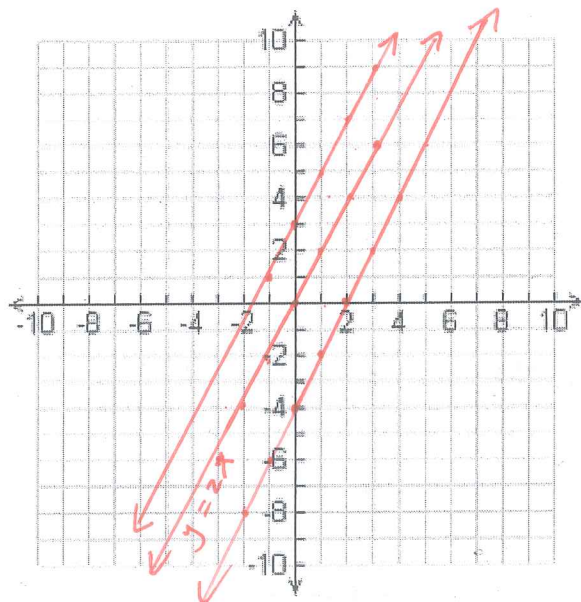> $y = f(x - h) + k$ or, equivalently, of $y - k = f(x - h)$
>
> is a translation horizontally h units and vertically k units.

Translating up or down: Graph below, the line $y = 2x$. Then draw a parallel line, 3 units higher.

    a. What is the new equation?

$$y = 2x + 3$$

    b. If $f(x) = 2x$, what is the equation of the new line in terms of $f(x)$?

$$y = f(x) + 3$$

    c. Graph another parallel line to $y = 2x$ but shifted 4 units down. What is the new equation?

$$y = 2x - 4$$

    d. If $f(x) = 2x$, what is the equation of the new line in terms of $f(x)$?

$$y = f(x) - 4$$

> Notice when the k is on the opposite side of the equation of the y, that the k tells you how the graph has been moved up or down.

Example 1: Given the functions below, describe the translation (how many units shifted up/down)

a. $y = f(x) + 10$

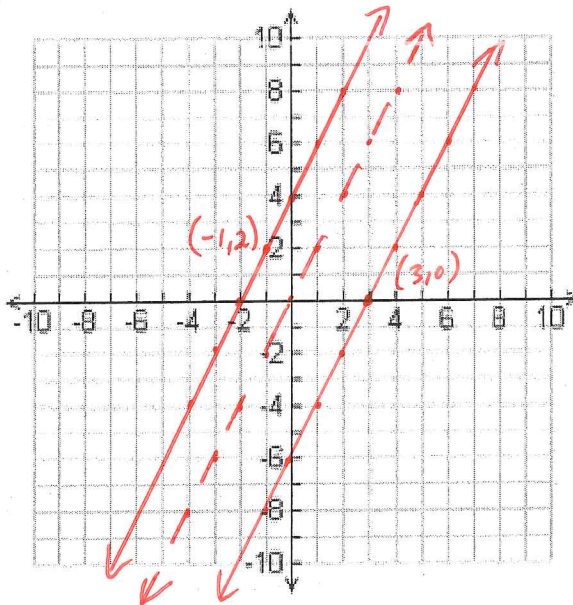VERTICAL TRANSLATION UP 10

b. $y = f(x) - 4.5$

VERTICAL TRANSLATION DOWN 4.5

c. $y = f(x) + 1$

VERTICAL TRANSLATION UP 1

Translating right or left: The h indicates a translation right/left.

Graph the equation $y = 2x$ below in a dotted line. Now graph a line parallel to that but shifted three units to the right.

Write the equation of your new line in point-slope form. Then write an equation of the new line in terms of $f(x)$.

$(3,0)$
$m = 2$

$y = 0 + 2(x - 3)$
$y = 2(x - 3)$

$y = f(x - 3)$

Plot a point 1 unit left and 2 units up from the origin. Draw a line parallel to the original line through this point and use the point to write an equation in point-slope form for the new line. Then write the equation in terms of $f(x)$.

$(-1, 2)$
$m = 2$

$y = 2 + 2(x - -1)$
$y = 2 + 2(x + 1)$
or
$y = 2(x + 1) + 2$

$y = f(x + 1) + 2$

If you move every point on the function $y = f(x)$ to a new point up k units and right h units, what is the equation of the translated function?

$y = f(x - h) + k$

☺ Chapter 4 Notes ☺

Notice that the k value is the opposite of how the graph moves right/left.

Example 2: Given the functions below, describe the translation (units right/left)

a. $y = f(x+6)$

HORIZONTAL TRANSLATION LEFT 6

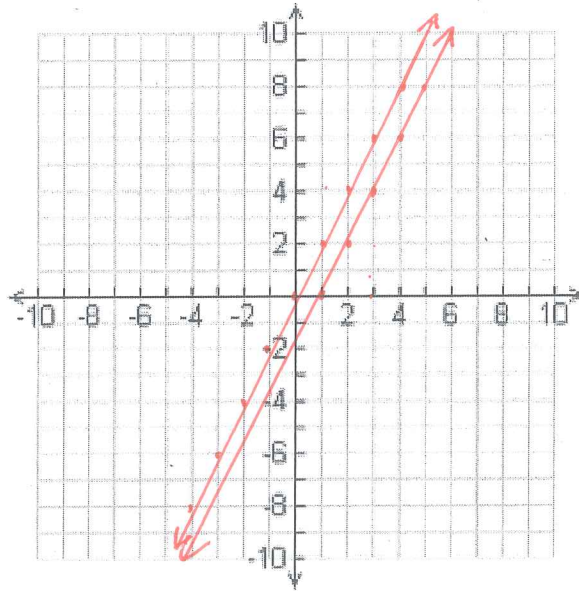b. $y = h(x-8)$

HORIZONTAL TRANSLATION RIGHT 8

c. $y = f(x+19)$

HORIZONTAL TRANSLATION LEFT 19

Example 3: Describe how the graph of $f(x) = 4 + 2(x-3)$ is a translation of the graph of
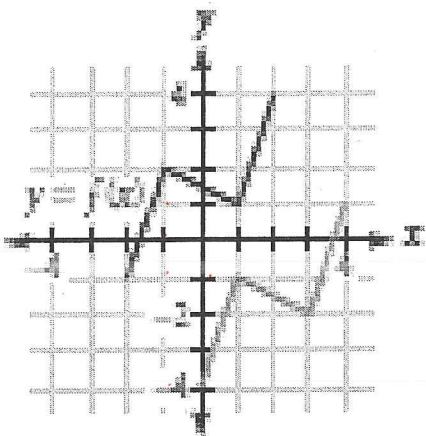
$f(x) = 2x.$

HORIZONTAL TRANSLATION RIGHT 3
VERTICAL TRANSLATION UP 4

b. Graph both lines on the graph below:

Example 4: The lower graph is a translation of the graph of function f. Write an equation for the red function in terms of $f(x)$.

DOWN 3   RIGHT 2

$$y = f(x-2) - 3$$